Fourier and Minimal Bending Analysis of Postural Sway Area

FRANCE SEVŠEK
University of Ljubljana
University College of Health Studies
Poljanska 26a, 1000 Ljubljana
SLOVENIA
france.sevsek@vsz.uni-lj.si

Abstract: A new method is presented for calculation of the outline of the area of the centre of pressure (COP) trajectory (sway area) for subjects standing still on a force platform. At first the outline of the COP area is determined by detecting the points that are furtherest from the centre in a given angular interval. To this outline a Fourier series is fitted by minimising the function that consists of squares of the differences between the calculated and measured distances from the centre, to which the bending energy and linear terms are added. This results in a simple, smooth outline which is predominantly outside the experimental COP area. The procedure has been successfully tested with simulated and clinical data. It proved to be fast, simple to implement and offered great versatility by choosing the appropriate bending and linear weighting constants.

Key–Words: stabilometry, force platform, sway area, Fourier analysis, outline bending

1 Introduction

Measurement of the centre of pressure (COP) movement with a force platform (stabilometry) is a standard procedure for assessment of postural stability in elderly and during rehabilitation. A subject stands still on a special platform that is mounted on pressure sensors transmitting data via analogue to digital converter to a computer. With a suitable software the time dependence of the trajectory of COP (sway) can be monitored.

As the human balance control system depends on feedback from the somatosensory, vestibular and visual systems, stabilometry can give clues about their functioning. It was shown that somatosensory function declines with age[1], diabetic neuropathy and often with stroke[2], resulting in diminished motor performance. In these cases it was suggested that introduction of input noise by vibrating insoles can improve balance control[3, 4]. An intensive research effort in stabilometry resulted also in developing quantitative models that take into account integration of various sensory inputs in postural control[5].

From the measured COP trajectory simple statistical parameters related to the distance and velocity of COP are usually determined. Quite often it is also of interest to compare the areas within which the movement of COP is confined[6]. In this case the principal component analysis (PCA) of the covariant matrix may be used.[7] Here the eigenvalues (σ_0^2) are calculated from the covariant matrix σ_{xy}^2:

$$\sigma_{xy}^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \overline{x})(y_i - \overline{y}), \tag{1}$$

where $\overline{x}$ and $\overline{y}$ are the mean values and the summation is done over all N measured points.

The two eigenvalues are thus

$$\sigma_0^2 = \left(\sigma_{xx}^2 + \sigma_{yy}^2 \pm \sqrt{(\sigma_{xx}^2 - \sigma_{yy}^2)^2 + 4(\sigma_{xy}^2)^2}\,\right)/2. \tag{2}$$

The sway area may be then reproduced by an ellipse with the two principal axes $1.96\sigma_0$ at the angle θ [7]:

$$\tan\theta = \frac{\sigma_{xy}^2}{\sigma_0^2 - \sigma_{yy}^2}. \tag{3}$$

Sometimes it is also convenient to analyse the stabilometric data in terms of concepts related to statistical mechanics, such as random walk model[8].

Thus, for given conditions, it is very important to select the most appropriate analysis of the trajectory. In this paper a new method for the calculation of the outline of the COP movement area is presented. The outline of the COP area is determined by detecting the points that are furtherest from the centre in a given angular interval. To this outline Fourier series is fitted by minimising the characteristic function. It is constructed as the usual sum of the square differences of the distances from the centre to which the linear and outline bending terms are added.

Obtained Fourier coefficients are similar to the Fourier descriptors usually employed in shape recognition [9, 10, 11]. The difference is that our contour points are function of the angle rather than the distance along the contour path. Although other shape description measures, such as moments or even simple compactness, were sometimes equivalent to Fourier descriptors [12] our choice was motivated by the ease of interpretation of the results and the possibility of simple bending energy and asymmetric fitting, as described below.

The described procedure has been successfully tested with simulated and clinical data.

1.1 Methods

Experimental data were collected by a force platform (Kistler 9286AA) using Bioware software. Raw data were copied to a Linux server where a system for data analysis had been developed. Such central data processing greatly simplified software maintenance and development. The user interface was written in PHP using Apache web server. It controls user logins, data uploads and calls shell scripts and specially developed programs for data analysis and manipulations.

The typical analysis of the stabilometry data starts by optional data smoothing by calculating moving average over chosen number of points. It then proceeds by plotting time and frequency distribution diagrams, and finishes by calculating areas and other parameters.

Most of the calculations were preformed on a Pentium IV computer running under Linux operating system. The programs were mostly written in Fortran and C whereas data plotting is done by the Gnuplot program.

By data simulations the portable pseudo-random number generator Ran3, based on a subtractive method, was used [13, 14]. It has a very long period and no evident defects. To eliminate the possible implementation defects the program was thoroughly statistically tested using standard methods [15].

1.2 Determination of the sway area contour

To determine the sway area contour all data points are converted into polar coordinates by calculating their distance R_i from the centre ($\overline{x}$, $\overline{y}$) and the respective polar angle ϕ_i. The full angle then is divided into chosen number of intervals, depending on the number of data points and required precision. For our measurements usually 50 intervals were sufficient. In each angular interval the point that is furtherest from the centre is determined. These points represent the first approximation for the sway area outline (Fig. 1). It must be noted that such an outline is uniquely defined for every selected angular value i.e. for every angle the radial vector from the centre crosses the outline only once.

In stabilometry we are usually not interested in detailed structure of the measured area but want to get some information about the region of support. This is the region where the COP could travel during the experiment while the subject maintained upright stance. This could be in principle obtained by prolonging the measuring time, but because of subject fatiguing effects such results would be of little use. A suitable approximation to the sway area is thus a region determined by a rather simple, mostly convex outline which is predominantly at the outer border of the area. As will be shown below, such an outline can be easily reproduced by Fourier analysis, considering also outline bending energy (Fig. 2).

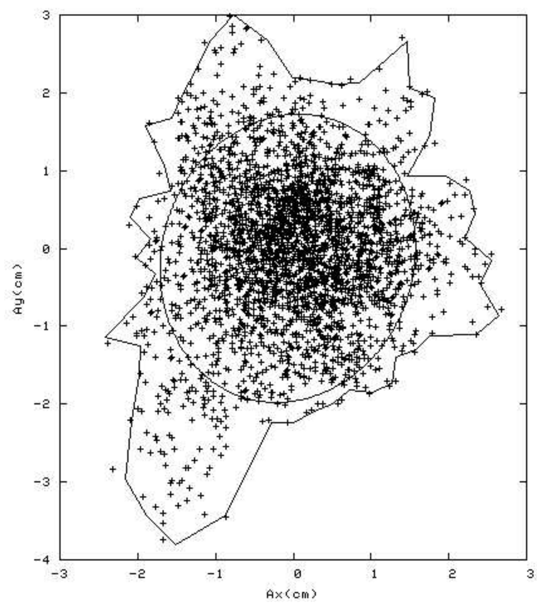

Fig. 1. An example of a measured sway area with outline determined by 50 points. The ellipse was determined by principal component analysis. The subject was 65 year old female, regularly exercising, standing on compliant, flat surface with open eyes.

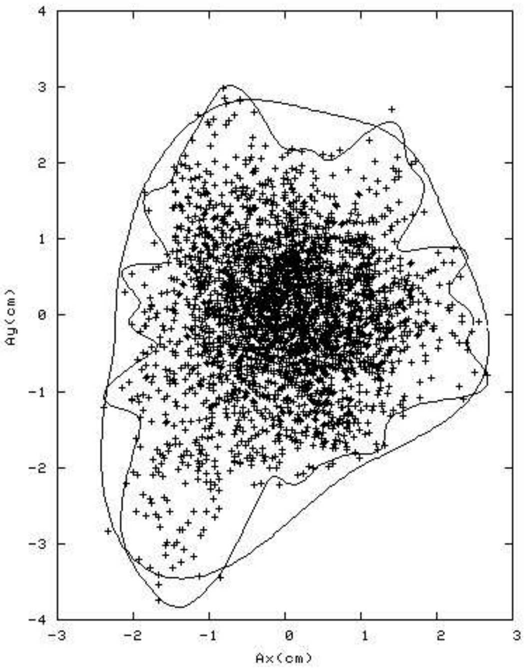

Fig. 2. The same experimental data as in Fig. 1, fitted with Fourier coefficients up to $m = 10$, as described in text, without bending (inner corrugated outline) and with $\omega = 0.1$ and $\gamma = 0.001$ (outer smooth outline).

1.3 Fourier coefficients of the contour

The smooth sway area outline can be conveniently expressed in polar coordinates $R(\phi)$, where R is the distance from the chosen origin of the coordinate system to the outline point at a given polar angle ϕ[16].

$$R(\phi) = R_0 + \sum_{m=1}^{m_{max}} [A_m cos(m\phi) + B_m sin(m\phi)], \quad (4)$$

where A_m and B_m are the appropriate Fourier coefficients and m_{max} the maximal number of coefficients used to describe the outline. The more coefficients are chosen, the smaller details of the shape can be reproduced.

There are various methods to obtain Fourier coefficients from the determined sway area outlines. Since we wanted to include in fitting procedure also the bending energy and linear term we decided for the most straightforward method - minimising the function (F):

$$F = \sum_{i=1}^{N} [R(\phi) - R_i]^2 - \omega R_{av} \sum_{i=1}^{N} [R(\phi) - R_i] + + \gamma \sum_{m=1}^{m_{max}} m^2 (m-1)^2 [A_m^2 + B_m^2]. \quad (5)$$

The first term is the usual sum of the squares of the differences between the calculated and experimental points. The second, linear, term takes care for slight asymmetry in fitting by preferring the points that are further from centre than the experimental ones if ω is positive. It was multiplied by the average value R_{av} so that the constant ω does not depend on outline size.

The last term in eq.(5) is related to the outline bending energy. It is constructed similarly to the bending energy of thin membrane of a vesicle which is in three dimensions [17]:

$$W_B = \frac{k_c}{2} \oint (C_1 + C_2)^2 dA, \quad (6)$$

where k_c stands for the membrane bending modulus, C_1 and C_2 are the principal curvatures of the membrane, and the integration is done over its neutral surface. If the curvatures are small only the second order terms may be considered, yielding the fourth order dependence in coefficients[18]. The form of bending energy must not depend on rotation of the coordinate system, thence $[A_m^2 + B_m^2]$ term, and must vanish for $m = 0$ and $m = 1$. The positive parameter γ determines the relative importance of this term. The larger it is, the more are higher m modes penalized in F and thus the simpler becomes the calculated outline. In the limiting case γ may be very large and the obtained outline becomes spherical, whereas at $\gamma = 0$ all modes are equally weighted and the calculated outline follows the experimentally determined one.

Fitting was done by minimising the function F of eq.(5) and considering $\frac{dF}{dA_m} = 0$ and $\frac{dF}{dB_m} = 0$.

$$\begin{aligned} \frac{dF}{dA_m} &= 2\sum_{i=1}^{N} [R(\phi) - R_i] \cos m\phi - \\ &- \omega R_{av} \sum_{i=1}^{N} \cos m\phi + \\ &+ 2\gamma \sum_{m=1}^{m_{max}} m^2 (m-1)^2 A_m = 0 \quad (7) \\ \frac{dF}{dB_m} &= 2\sum_{i=1}^{N} [R(\phi) - R_i] \sin m\phi - \\ &- \omega R_{av} \sum_{i=1}^{N} \sin m\phi + \\ &+ 2\gamma \sum_{m=1}^{m_{max}} m^2 (m-1)^2 B_m = 0 \quad (8) \end{aligned}$$

If we also use the expansion of $R(\phi)$ as given by eq.(4), these relations give a system of equations that is represented by a matrix equation where the left hand side is a of the type $\alpha_{mk}X_mX_k$, whereas the right hand side is β_mX_m with X_m standing for A_m or B_m.

Such a system can be easily solved by the method of LU decomposition [14]. It decomposes the matrix into the product of a lower and an upper triangular one from which the solutions can be calculated by a simple substitution.

1.4 Simulated data

Our procedure was tested by simulated and clinical data. Simulated data are advantageous for testing as their shape is well defined and the results are known in advance. But they must be as similar as possible to the experimental ones. For this reason our data were calculated by considering completely free random movement of the COP within a chosen ellipsoidal region and with Boltzmann distribution outside. For this purpose random walk procedure was used combined with Metropolis algorithm[19]. In each step the COP position was randomly moved. If the resulting position was within the chosen ellipsoidal region it was accepted and the procedure was repeated. But, when the resulting position was outside the region and the previous one was inside, the distance (ΔR) from the border was calculated. Such a move was accepted only with the probablility $e^{-(\Delta R)^2/T}$, where $(\Delta R)^2$ plays the role of energy and is proportional to the square of the distance ($(\Delta R)^2$), whereas T corresponds to the temperature. The procedure was similar also in the case when the previous point was outside, too. In this case the calculated value ΔR was the difference between the distances from the area centre of the two points. If ΔR was negative, the move was accepted, otherwise it was accepted only with the probability $e^{-(\Delta R)^2/T}$.

The procedure started with a point somewhere inside the ellipsoidal region and the first few thousand points were rejected to allow the system to thermalize.

This description is equivalent to the movement of a particle in a potential which is flat in the central ellipsoidal region and quadratic outside. In such a way COP can move outside the chosen region, but the probability of finding it outside decreases with distance from the boundary whereas parameter T defines this probability.

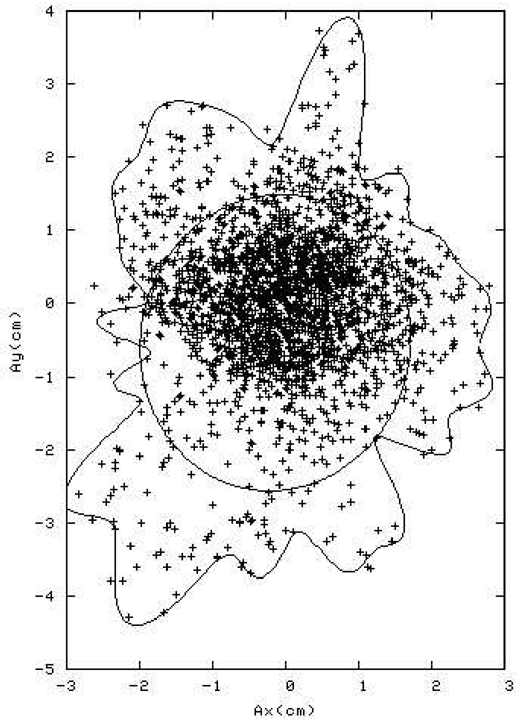

Fig. 3. Simulated data for $T = 0.5$ and the ellipsoidal region with $a = 1.0$ and $b = 0.5$ fitted with with Fourier coefficients up to $m = 10$. The inner ellipsoidal outline was determined by the principal component analysis.

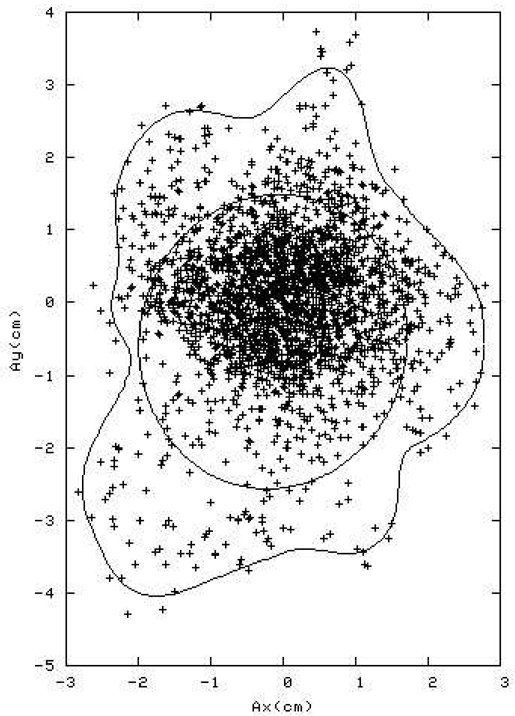

Fig. 4. The same data as in Fig. 3 fitted with $\omega = 0$ and $\gamma = 10^{-4}$.

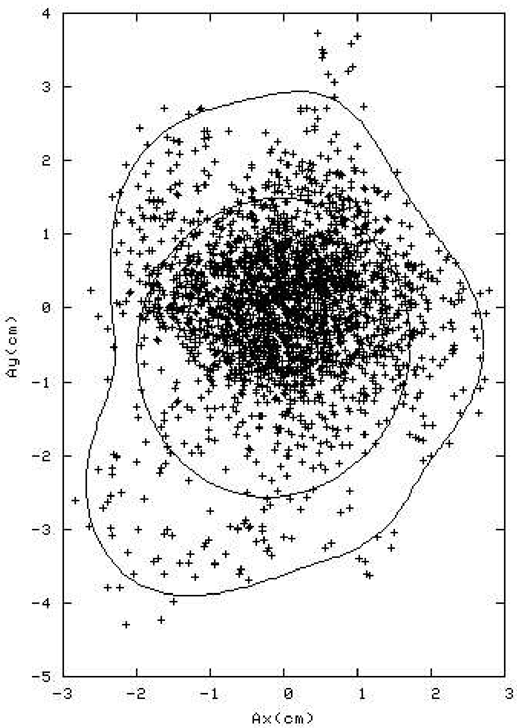

Fig. 5. The same data as in Fig. 3 fitted with $\omega = 0$ and $\gamma = 10^{-3}$.

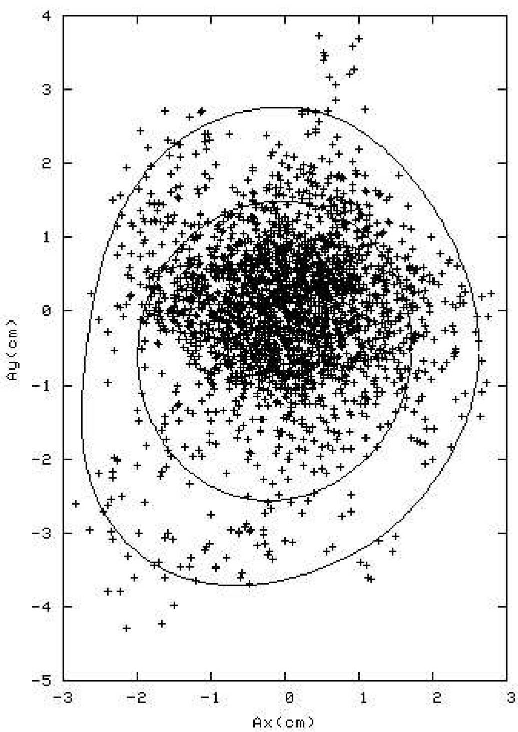

Fig. 6. The same data as in Fig. 3 fitted with $\omega = 0$ and $\gamma = 10^{-2}$.

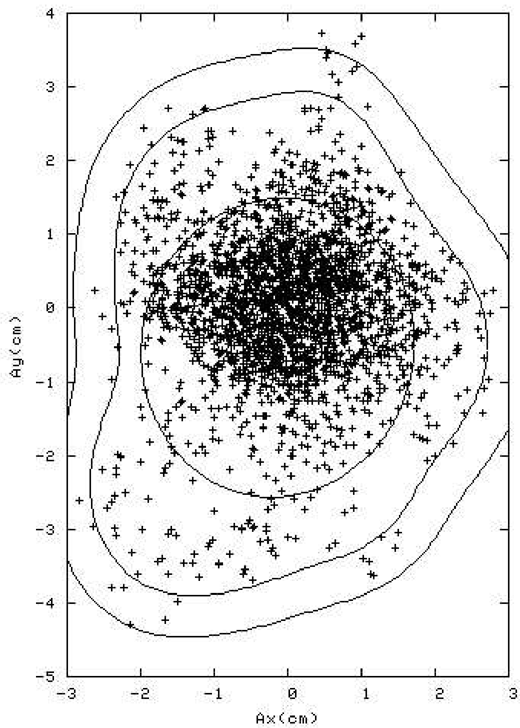

Fig. 7. The same data as in Fig. 5 fitted with $\gamma = 10^{-3}$ and $\omega = 0$. The outer outline was obtained by setting $\omega = 0.2$.

2 Results and Discussion

An example of the simulated data are shown in Figs. 3 to 7 together with the outlines calculated by the described procedure for different values of the bending weight parameter (γ). It is seen that the calculated outline gets more spherical as the parameter γ increases. To sufficiently describe the experimental data it was found that the values of about $\gamma = 10^{-3}$ can be used. Here described procedure results in a nearly convex outline as required for the interpretation of experimental data. This method is much simpler and formally correct than the procedure of elimination of concave points in the outline[6].

The inner ellipsoids in all shown figures were calculated by the principal component analysis. As expected this gives smaller area as it encompasses only 85.35 % of all points when the distribution is normal[7].

The influence of linear term weight (ω) is evident from Fig. 7. As expected from eq.(5) increasing ω moves the calculated outline further away from the experimentally determined COP region. In contrast to asymmetric fitting [6] this method is much simpler, faster and gives quite similar results.

3 Conclusion

It was shown that Fourier analysis of the sway area contour is very suitable for data interpretation. It gives not only the value of the sway area but also some information about its shape. Although this method is limited to the shapes that have a uniquely defined contour as a function of polar angle, this is not a limitation in real situations where the movement of COP over supporting surface is studied. Namely, here nearly convex sway area contour is usually of interest. Thus, expressing the outline in terms of Fourier coefficients proved to be very suitable for determination of outlines with reduced bending and situated predominantly outside the sway area.

All the described computer programs are available from the author upon request.

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