Determination of Sway Area by Fourier Analysis of its Contour

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Abstract: A new method is presented for the calculation of the area of the centre of pressure (COP) trajectory (sway area) for subjects standing still on a force platform. The outline of the COP area is determined by detecting the points that are furtherest from the centre in a given angular interval. To this outline a Fourier series is fitted so that the points inside and outside are weighted differently. The procedure has been successfully tested with simulated and clinical data.

Key–Words: stabilometry, force platform, sway area, Fourier analysis

1 Introduction

Measurement of the centre of pressure (COP) movement with a force platform (stabilometry) is a standard procedure for assessment of postural stability during rehabilitation. A subject stands still on a special platform that is mounted on pressure sensors transmitting data via analogue to digital converter to a computer. With a suitable software the time dependence of the trajectory of COP (sway) can be monitored.

As the human balance control system depends on feedback from the somatosensory, vestibular and visual systems, stabilometry can give clues about their functioning. It was shown that somatosensory function declines with age[1], diabetic neuropathy and often with stroke[2], resulting in diminished motor performance. In these cases it was suggested that introduction of input noise by vibrating insoles can improve balance control[3, 4]. An intensive research effort in stabilometry resulted also in developing quantitative models that take into account integration of various sensory inputs in postural control[5].

From the measured COP trajectory simple statistical parameters related to the distance and velocity of COP are usually determined. Quite often it is also of interest to compare the areas within which the movement of COP is confined. In this case the principal component analysis (PCA) of the covariant matrix may be used.[6] Here the eigenvalues (σ_0^2) are calculated from the covariant matrix σ_{xy}^2:

$$\sigma_{xy}^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \overline{x})(y_i - \overline{y}), \tag{1}$$

where $\overline{x}$ and $\overline{y}$ are the mean values and the summation is done over all N measured points.

The two eigenvalues are thus

$$\sigma_0^2 = \left(\sigma_{xx}^2 + \sigma_{yy}^2 \pm \sqrt{(\sigma_{xx}^2 - \sigma_{yy}^2)^2 + 4(\sigma_{xy}^2)^2}\,\right)/2. \tag{2}$$

The sway area may be then reproduced by an ellipse with the two principal axes $1.96\sigma_0$ at the angle θ [6]:

$$\tan\theta = \frac{\sigma_{xy}^2}{\sigma_0^2 - \sigma_{yy}^2}. \tag{3}$$

Sometimes it is also convenient to analyse the stabilometric data in terms of concepts related to statistical mechanics, such as random walk model[7].

Thus, for given conditions, it is very important to select the most appropriate analysis of the trajectory. In this paper a new method for the calculation of the area of the COP movement is presented. The outline of the COP area is determined by detecting the points that are furtherest from the centre in a given angular interval. To this outline Fourier series is fitted by minimising the characteristic function. It is constructed as the sum of the square differences of the distances from the centre where the terms were differently weighted depending whether the calculated point is inside or outside the experimental contour. Obtained Fourier coefficients are similar to the Fourier descriptors usually employed in shape recognition [8, 9, 10]. The difference is that our contour points are function of the angle rather than the distance along the contour path. Although other shape description measures, such as moments or even simple compactness, were sometimes equivalent to Fourier descriptors [11] our choice was motivated by the ease of interpretation of the results and the possibility of simple asymmetric fitting, as described below.

The described procedure has been successfully tested with simulated and clinical data.

1.1 Methods

Experimental data were collected by a force platform (Kistler 9286AA) using Bioware software. Raw data were copied to a Linux server where a system for data analysis had been developed. Such central data processing greatly simplified software maintenance and development. The user interface was written in PHP using Apache web server. It controls user logins, data uploads and calls shell scripts and specially developed programs for data analysis and manipulations. The programs were mostly written in Fortran whereas data plotting is done by the Gnuplot program.

The typical analysis of the stabilometry data starts by optional data smoothing by calculating moving average over chosen number of points, proceeds by plotting time and frequency distribution diagrams, and finishes by calculating areas and other parameters.

1.2 Determination of the sway area contour

To determine the sway area contour all data points are converted into polar coordinates by calculating their distance R_i from the centre ($\overline{x}$, $\overline{y}$) and the respective polar angle ϕ_i. The full angle then is divided into chosen number of intervals, depending on the number of data points and required precision. For our measurements usually 50 intervals were sufficient. In each angular interval the point that is furtherest from the centre is determined. These points represent the first approximation for the sway area outline (Fig. 1). It must be noted that such an outline is uniquely defined for every selected angular value i.e. for every angle the radial vector from the centre crosses the outline only once.

In stabilometry we are usually not interested in detailed structure of the measured area but want to get some information about the surface of support. This is the surface where the COP could travel during the experiment while the subject maintained upright stance. This could be in principle obtained by prolonging the measuring time, but because of subject fatiguing effects such results would be of little use. A suitable approximation to the sway area is thus a region determined by a convex outline. For this purpose the contour was scanned for concave points and these were replaced by the average values of their neighbouring points. The procedure was repeated until no concave contour points were left or the number of repetitions reach predetermined value, which was usually set to a rather low, about 10. In this case especially more complicated contours quite often still consisted of some concave regions, but these were smooth and small (Fig. 2).

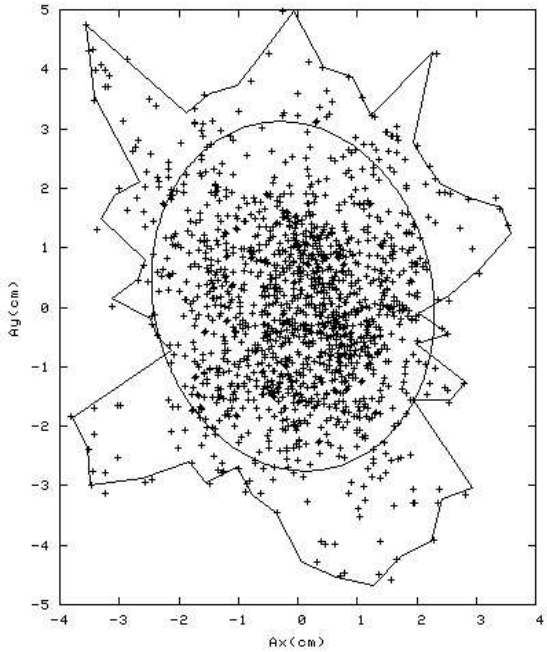

Fig.1 An example of a measured sway area with outline determined by 50 points. The ellipse was determined by PCA.

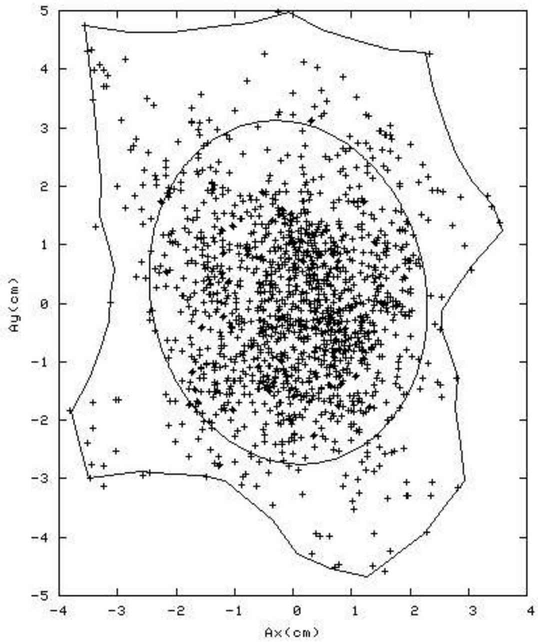

Fig.2 An example of a measured sway area with a nearly convex outline determined obtained from data shown in Fig.1. The ellipse was determined by PCA.

1.3 Fourier Coefficients of the contour

The smooth sway area outline can be conveniently expressed in polar coordinates $R(\phi)$, where R is the distance from the chosen origin of the coordinate system to the contour point at a given polar angle ϕ[12].

$$R(\phi) = R_0 + \sum_{m=1}^{m_{max}} [A_m cos(m\phi) + B_m sin(m\phi)], \quad (4)$$

where A_m and B_m are the appropriate Fourier coefficients and m_{max} the maximal number of coefficients used to describe the contour. The more coefficients are chosen, the smaller details of the shape can be reproduced.

There are various methods to obtain the Fourier coefficients from the determined sway area outlines. Since our contour points need not be equidistant and computational time is not crucial, we use the most straightforward method - least square fitting of Eq.4 to the determined contour points. Here, the sum of the squares of the differences between the calculated and experimental points is usually minimised. But in the case of stabilometry maximal area is of interest. Thus we decided to use asymmetric fitting function as follows:

For every experimental point the distance from the origin ρ_i and the polar angle ϕ_i are calculated and the corresponding contour point (R_i) is determined from eq.(4). The square of the difference between R_i and ρ_i is calculated. It is multiplied by a predetermined constant (ω) if the calculated point is closer to the origin than the experimental one. This value is added to the characteristic function (χ^2) and summation is performed over all N contour points.

$$\chi^2 = \sum_{i=1}^{N} \omega_i [R_i - \rho_i]^2, \quad (5)$$

where

$$\omega_i = \begin{cases} 1 \quad for \quad R_i > \rho_i, \\ \omega \quad for \quad R_i \leq \rho_i. \end{cases} \quad (6)$$

When $\omega = 1$ the fitting is symmetric, whereas $\omega > 1$ decreases the distance the fitted curve can penetrate inside the contour.

Fitting was done by minimising the function χ^2. For symmetric case normal equations were solved by the method of LU decomposition [13]. It decomposes the matrix into the product of a lower and an upper triangular one from which the solutions can be calculated by a simple substitution. The fitting of asymmetric case was more time consuming and was done by simplex method[13].

1.4 Simulated data

Our procedure was thoroughly tested by simulated and clinical data. Simulated data are advantageous for testing as their shape is well defined and the results are known in advance. But they must be as similar as possible to the experimental ones. For this reason our data were calculated by considering completely free random movement of the COP within a chosen ellipsoidal region with soft boundary. This means that COP was able to move outside the boundary, but this position was accepted only with the probability $e^{-E/T}$, where E plays the role of energy and is proportional to the square of the distance from the boundary, whereas T corresponds to the temperature. This description is equivalent to the movement of a particle in a potential which is flat in the central ellipsoidal region and quadratic outside. In such a way COP can move outside the chosen region, but the probability of finding it outside decreases with distance from the boundary whereas parameter T defines this probability.

2 Results and Discussion

An example of the simulated data are shown in Fig.3 together with the contours calculated by Fourier and principal component analysis. As expected PCA gives smaller area as it encompasses only 85.35 % of all points when the distribution is normal[6].

It was also of interest to compare the calculated areas for both methods as a function of parameter T (temperature). As seen from Fig.4 at low temperatures the movement of COP is confined to the basic ellipse and both methods give similar results which correspond to the area of the basic ellipse. Increasing the temperature means more vigorous movement of COP and more of its time is spent outside the ellipsoidal region. Both areas thus increase with temperature, but PCA gives allways smaller values.

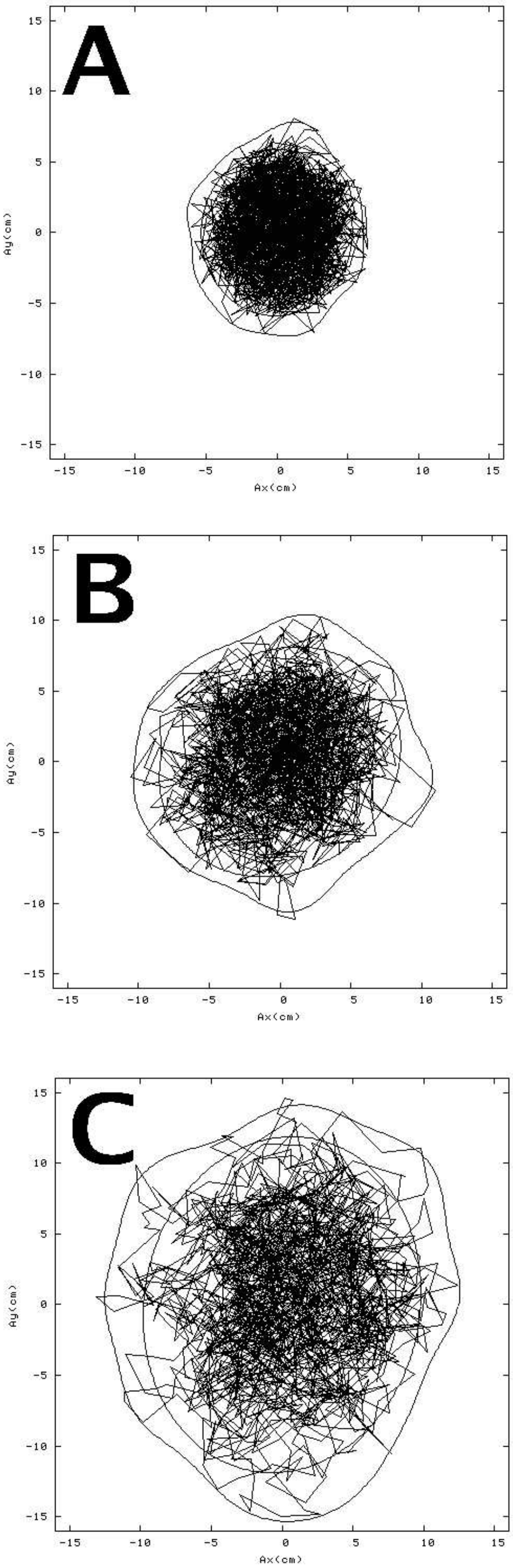

Fig.3 Simulated data for soft boundary ellipse (a = 4 cm, b = 3 cm, $\theta = 80^0$) for T = 0.5 (A), 2.0 (B) and 5.0 (C). The outer and inner contours were determined by Fourier and principal component analysis, respectively.

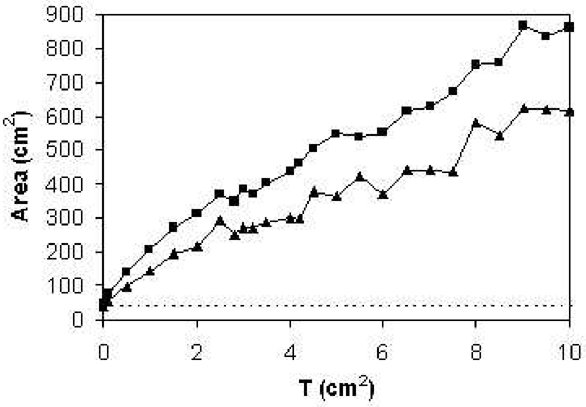

Fig.4 Areas of simulated data for soft boundary ellipse (a = 4 cm, b = 3 cm) as a function of temperature (T). Upper curve was calculated by Fourier analysis of convex surface, lower one is the result of PCA and dashed line shows the area of the basic ellipse.

3 Conclusion

It was shown that Fourier analysis of the sway area contour is very suitable for data interpretation. It gives not only the value of the sway area but also some information about its shape. Although this method is limited to the shapes that have a uniquely defined contour as a function of polar angle this is not a limitation in real situations where the movement of COP over supporting surface is studied. It was argued that here nearly convex sway area contour is of interest. Besides, expressing the contour as a function of polar angle proved to be very suitable for asymmetric fitting since the calculated contour should cross into the sway area as little as possible.

All the described computer programs are available from the author upon request.

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